# Math Circles

## Conway's Rational Tangles

# Let’s Think!

- Who is John Conway?
- What are tangles?
- What does the word `rational’ mean?

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- Who is John Conway?
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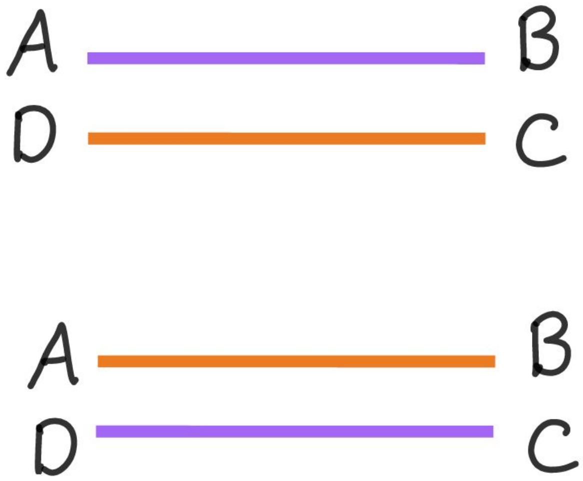
**Fractions and knots!**

# Operations:

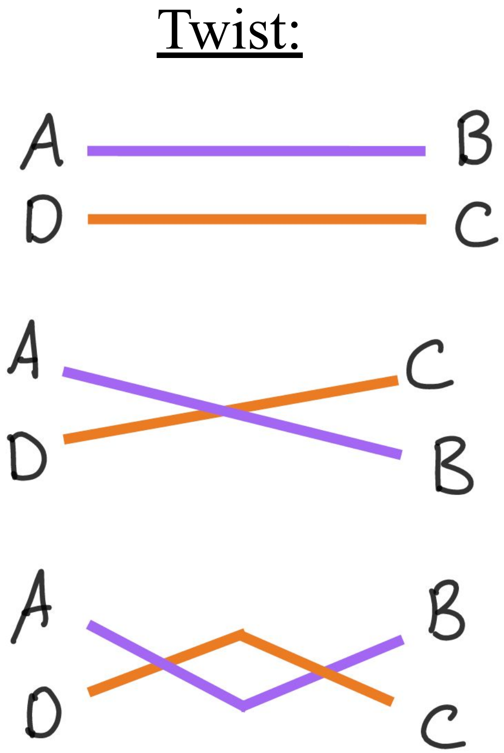
- Initial Position:
  - Lay the ropes parallel to one another.
  - Any time the ropes are parallel to each other, we call this our "initial position."

Initial Position:



# Operations: Twist

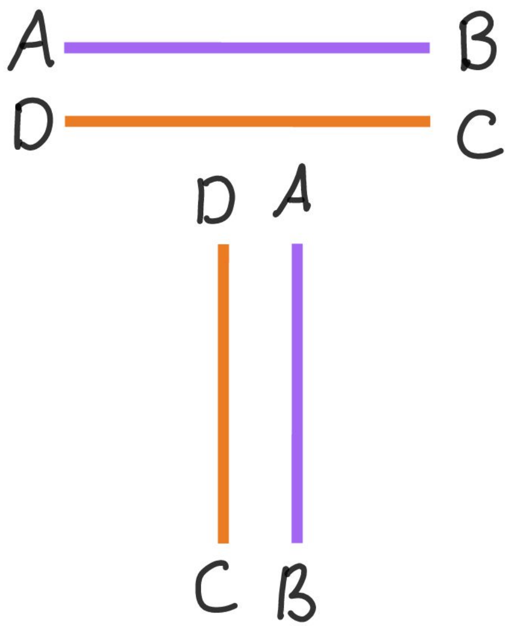
- Twist:
  - Position B raises the rope
  - Position C ducks under the rope



# Operations: Rotate

- <u>Rotate:</u>
  - Move one position to the left (Clockwise)

<u>Rotate</u>



# Shorthand:

- T: represents a `Twist’
- R: represents a `Rotate’

Examples:

Two twists: TT

Twist, Twist, Rotate: TTR

# Where does the Math come in?

- **Initial position,** two parallel ropes, denotes zero position: **0**
- **Twist:** We are going to give <u>Twist</u> the mathematical property of: **adding 1.**

<u>Example:</u>

Twist: $0 + T \Rightarrow 0 + 1$ ……. We can write as: $0 \xrightarrow{T} 1$

Two Twist: $0+T+T \Rightarrow 0+1+1$ ……. We can write as: $0 \xrightarrow{TT} 2$

# Where does the Math come in?

- What about rotations???
- Let’s try to undo a twist!

# Where does the Math come in?

- We have: T

$$0 \xrightarrow{T} 1 \xrightarrow{R} \_ \xrightarrow{T} 0$$

- What mathematical operation of **R** causes a 1 to become a -1?

# Where does the Math come in?

- We have: T

$$0 \xrightarrow{T} 1 \xrightarrow{R} \_ \xrightarrow{T} 0$$

- What mathematical operation of **R** causes a 1 to become a -1?

$$0 \xrightarrow{T} 1 \xrightarrow{R} -1 \xrightarrow{T} 0$$

# Where does the Math come in?

- We have: TT

$$0 \xrightarrow{TT} \_ \xrightarrow{R} \_ \xrightarrow{T} \_ \xrightarrow{R} \_ \xrightarrow{TT} 0$$

- What mathematical operation of **R** gets us back to 0 (initial position)?

# Where does the Math come in?

- We have:

$$0 \xrightarrow{TT} \_\_ \xrightarrow{R} \_\_ \xrightarrow{T} \_\_ \xrightarrow{R} \_\_ \xrightarrow{TT} 0$$

- What mathematical operation of **R** gets us back to 0 (initial position)?

$$0 \xrightarrow{TT} \underline{2} \xrightarrow{R} \underline{-1/2} \xrightarrow{T} \underline{1/2} \xrightarrow{R} \underline{-2} \xrightarrow{TT} 0$$

# Tangle Numbers

Find the tangle numbers to the following combination of twist and rotations:

1. Sequence: TTT
2. Sequence: TTRTTRTTTT

# Tangle Numbers

1. Sequence: $TTT$

   Answer: $\frac{0}{1} \xrightarrow{\text{T}} \frac{1}{1} \xrightarrow{\text{T}} \frac{2}{1} \xrightarrow{\text{T}} \frac{3}{1}$

2. $TTRTTRTTTT$

   Answer: $\frac{0}{1} \xrightarrow{\text{TT}} \frac{2}{1} \xrightarrow{\text{R}} \frac{-1}{2} \xrightarrow{\text{TT}} \frac{3}{2} \xrightarrow{\text{R}} \frac{-2}{3} \xrightarrow{\text{TTTT}} \frac{10}{3}$

# Let's Practice!

| Initial Position | Operations to return to the initial position |
|---|---|
| $T$ | |
| $TT = T^2$ | |
| $TTT = T^3$ | |
| $TTTT = T^4$ | |
| $T^n$ | |

# Let’s Practice!

| Initial Position | Operations to return to the initial position |
|---|---|
| $T$ | $RT$ |
| $TT = T^2$ | $RT\,RTT$ |
| $TTT = T^3$ | $RT\,RTT\,RTT$ |
| $TTTT = T^4$ | $RT\,RTT\,RTT\,RTT$ |
| $T^n$ | $RT(RTT)^{n-1}$ |

# Finale!

Sequence: TTRTRTRTRTTRTTTT

# Finale!

Sequence: TTRTRTRTRTTRTTTT

(b) Sequence: $TTRTRTRTRTTRTTTT$

Answer:

$$\frac{0}{1} \xrightarrow{\text{TT}} \frac{2}{1} \xrightarrow{\text{R}} \frac{-1}{2} \xrightarrow{\text{T}} \frac{1}{2} \xrightarrow{\text{R}} \frac{-2}{1} \xrightarrow{\text{T}} \frac{-1}{1} \xrightarrow{\text{R}} \frac{1}{1} \xrightarrow{\text{T}} \frac{2}{1} \xrightarrow{\text{R}} \frac{-1}{2} \xrightarrow{\text{TT}} \frac{3}{2} \xrightarrow{\text{R}} \frac{-2}{3} \xrightarrow{\text{TTTT}} \frac{10}{3}$$

# References


- Math Circles (by Kelly Barnes)
- Tom Davis. *Conway's rational tangles. 2010*
- James Tanton. *Understanding Rational Tangles. 2012*